MATH

OPERATIONS AND ALGEBRAIC THINKING

Write and interpret numerical expressions.

1. **Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.**

Evaluate the expression 2 {5[12 + 5(500 – 100) + 399]}

Students should have experiences working with the order of first evaluating terms in parentheses, then brackets, and then braces.

The first step would be to subtract 500 – 100 = 400. Then multiply 400 by 5 = 2,000.

Inside the bracket, there is now [12 + 2,000 + 399]. That equals 2,411.

Next multiply by the 5 outside of the bracket. 2,411 × 5 = 12.055.

Next multiply by the 2 outside of the braces. 12,055 × 2 = 24,110.

2. **Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.**
   For example, express the calculation "add 8 and 7, then multiply by 2" as 2(8 + 7). Recognize that 3(18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.

Write an expression for the steps "double five and then add 26".

Student: (2 × 5) + 26

Describe how the expression 5(10 × 10) relates to 10 × 10.

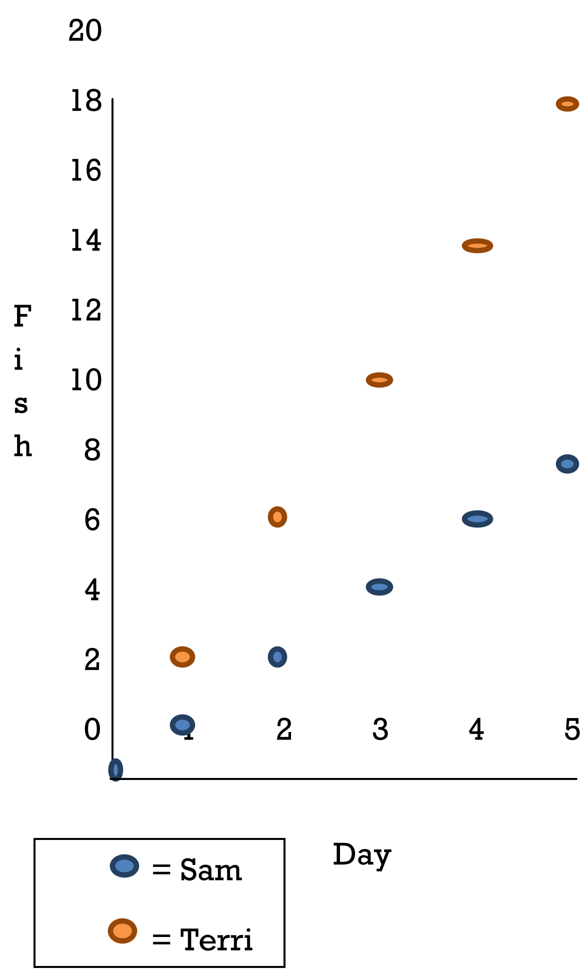
Student: The expression 5(10 × 10) is 5 times larger than the expression 10 × 10 since I know that 5(10 × 10) means that I have 5 groups of (10 × 10).

Analyze patterns and relationships.

Days	Sam's Total Number of Fish	Terri's Total Number of Fish
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri's fish is always greater. Terri's fish is also always twice as much as Sam's fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days?

CATCHING FISH



3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
For example, given the rule "Add 3" and starting number 0, and given the rule "Add 6" and the starting number 0,generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

NUMBER AND OPERATIONS IN BASE TEN

Understand the place value system.

1. **Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.**

In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.

5	5	.	5	5

The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.

2. **Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.**

Example:

$2.5 \times 10^3 = 2.5 \times (10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$

$350 \div 10^3 = 350 \div 1{,}000 = 0.350 = 0.35$

3. **Read, write, and compare decimals to thousandths.**
   a. **Read and write decimals to thousandths using base-ten numerals, number names, and expanded form.**
   b. **Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.**

Example:

$347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1{,}000)$

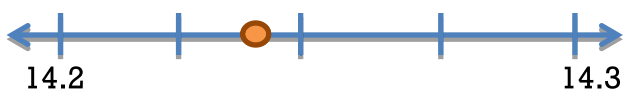
Example:

Comparing 0.25 and 0.17, a student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison.

Example:

Round 14.235 to the nearest tenth.

Students recognize that the possible answer must be in tenths, thus it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1: 225 × 12

I broke up 12 into 10 and 2.

225 × 10 = 2,250

225 × 2 = 450

2,250 + 450 = 2,700

Student 2: 225 × 12

I broke up 225 into 200 and 25.

200 × 12 = 2,400

I broke 25 up into 5 × 5, so I had 5 × 5 × 12 or 5 × 12 × 5.

5 × 12 = 60.   60 × 5 = 300.

I then added 2,400 and 300.

2,400 + 300 = 2,700

5. Fluently multiply multi-digit whole numbers using the standard algorithm.

6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Example:

There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created?

<u>Student 1</u>

1,716 divided by 16

There are 100 16's in 1,716.

1,716 – 1,600 = 116

I know there are at least 6 16's.

116 – 96 = 20

I can take out at least 1 more 16.

20 – 16 = 4

There were 107 teams with 4 students left over. If we put the extra students on different teams, 4 teams will have 17 students.

<u>Student 2</u>

1,716 divided by 16

There are 100 16's in 1,716.

Ten groups of 16 is 160. That's too big.

Half of that is 80, which is 5 groups.

I know that 2 groups of 16's is 32.

I have 4 students left over.

1716 - 1600	100
116 - 80	5
36 - 32	2
4	

Examples:

3.6 + 1.7

A student might estimate the sum to be larger than 5 because 3.6 is more than 3 ½ and 1.7 is more than 1 ½ .

5.4 – 0.8

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

6 × 2.4

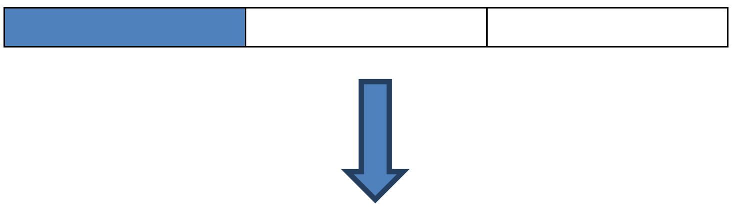
A student might estimate an answer between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 × 2 ½ and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (1/2 of a group of 6).

7. **Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.**

FRACTIONS

Use equivalent fractions as a strategy to add and subtract fractions.

Example: 1/3 + 1/6



1/3 is the same as 2/6

I drew a rectangle and shaded 1/3. I knew that if I cut every third in half, then I would have sixths. Based on my picture, 1/3 equals 2/6. Then I shaded in another 1/6 with red. I ended up with an answer of 3/6, which is equal to ½ .

1. **Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.**

   For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

2. **Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.***

Example:

Your teacher gave you 1/7 of the bag of candy. She also gave your friend 1/3 of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your answer?

Student 1

1/7 is really close to 0. 1/3 is larger than 1/7, but still less than ½ . If we put them together, we might get close to 1/2.

1/7 + 1/3 = 3/21 + 7/21 = 10/21. The fraction does not simplify. I know that 10 is half of 20, so 10/21 is a little less than ½ .

Another example: 1/7 is close to 1/6 but less than 1/6, and 1/3 is equivalent to 2/6, so I have a little less than 3/6 or ½ .

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. **Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b)*. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.**
For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people, each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Example:

Your teacher gives 7 packs of paper to your group of 4 students. If you share the paper equally, how much paper does each student get?

Student 1 Student 2 Student 3 Student 4

Pack1 Pack 2 Pack 3 Pack 4

Pack 5 Pack 6 Pack 7

Each student receives 1 whole pack of paper and $\frac{1}{4}$ of each of the 3 packs of paper. So each student gets 1 $\frac{3}{4}$ packs of paper.

Example:

Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

Student:

I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals ½ .

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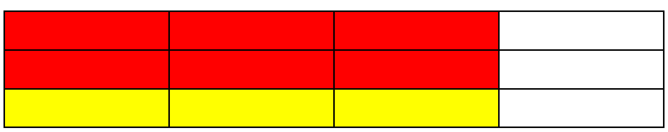
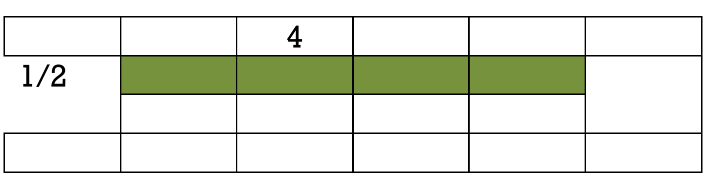


1/3
1/3
1/3

The home builder needs to cover a small storage room floor with carpet. The storage room is 4 meters long and half of a meter wide. How much carpet do you need to cover the floor of the storage room? Use a grid to show your work and explain your answer. In the grid below I shaded the top half of 4 boxes. When I added them together, I added ½ four times, which equals 2. I could also think about this with multiplication: ½ × 4 is equal to 4/2 which is equal to 2.



4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. **Interpret multiplication as scaling (resizing) by:**
    a. **Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.**

Example:

How does the product of 225 × 60 compare to the product of 225 × 30? How do you know?

Since 30 is half of 60, the product of 225 × 60 will be double or twice as large as the product of 225 × 30.

    b. **Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a/b = (n × a)/(n × b)* to the effect of multiplying *a/b* by 1.**

Example:

2 2/3 × 8 must be more than 8 because 2 groups of 8 is 16 and 2 2/3 is almost 3 groups of 8. So the answer must be close to, but less than 24.

¾ = (5 × 3)/(5 × 4) because multiplying ¾ by 5/5 is the same as multiplying by 1.

6. **Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.**

Example:

There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many buses would it take to carry *only* girls?

Student: 2 ½ × 2/5 =

I split the 2 ½ into 2 and ½.

2 × 2/5 = 4/5

½ × 2/5 = 2/10

I then added 4/5 and 2/10.

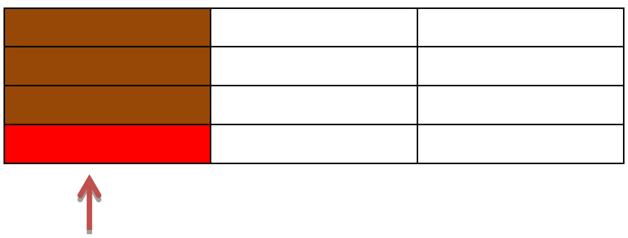
That equals 1 whole bus load.

Example:

Four students sitting at a table were given 1/3 of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?

The diagram shows the 1/3 pan divided into 4 equal shares with each share equaling 1/12 of the pan.

1/3

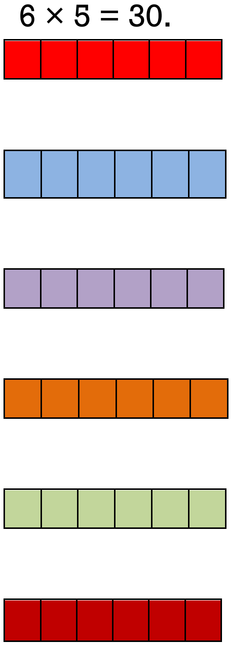


1/12

The bowl holds 5 liters of water. If we use a scoop that holds 1/6 of a liter, how many scoops will we need in order to fill the entire bowl?

Student:

I created 5 boxes. Each box represents 1 liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 × 5 = 30.



1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6

A whole has 6/6, so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 = 30/6

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

b. Interpret division of a whole number by a unit fraction, and compute such quotients. ***For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.***

c. **Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally?***

Example:

How many 1/3-cup servings are in 2 cups of raisins?

Student:

I know that there are three 1/3 cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by $1/3 = 2 \times 3 = 6$ servings of raisins.

MEASUREMENT AND DATA

Convert like measurement units within a given measurement system.

1. **Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.**

Example: 100 cm = 1 meter

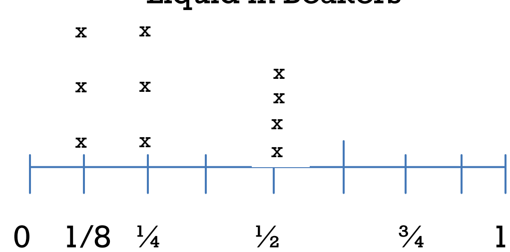
Feet	Inches
0	0
1	12
2	24
3	36

Represent and interpret data.

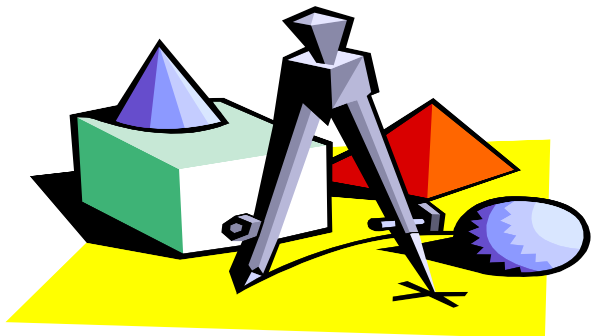
2. **Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.***

Ten beakers, measured in liters, are filled with a liquid.

Liquid in Beakers



Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.



3. **Recognize volume as an attribute of solid figures and understand concepts of volume measurement.**
   a. **A cube with side length 1 unit, called a unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.**
   b. **A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.**

4. **Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.**

5. **Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.**
   a. **Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.**
   b. **Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.**
   c. **Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.**

GEOMETRY

Graph points on the coordinate plane to solve real-world and mathematical problems.

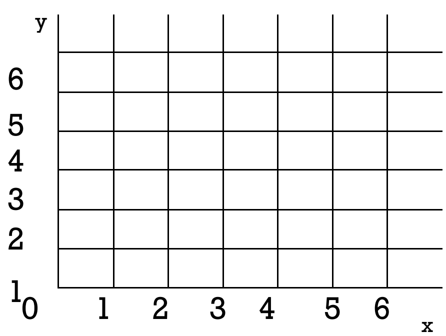
Example:

Connect these points in order on the coordinate grid below:

(2,2) (2,4) (2,6) (2,8) (4,5) (6,8) (6,6) (6,4) and (6,2).

What letter is formed on the grid?

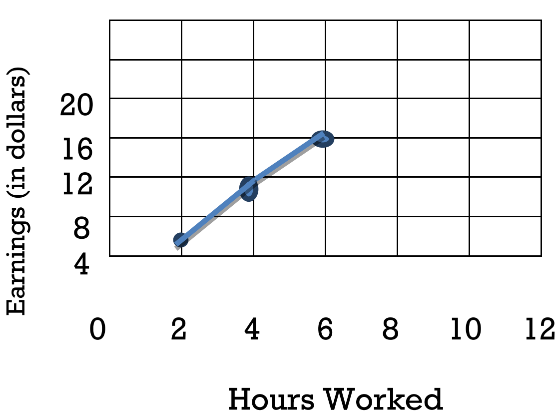
Solution: "M" is formed.



Example:

Use the graph below to determine how much money Jack makes after working exactly 9 hours.

Earnings and Hours Worked



1. **Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).**

2. **Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.**

Classify two-dimensional figures into categories based on their properties.

3. **Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.** ***For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.***

A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?

Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons.

All rectangles have 4 right angles. Squares have 4 right angles, so they are also rectangles. True or False?

A trapezoid has 2 sides parallel, so it must be a parallelogram. True or false?

4. **Classify two-dimensional figures in a hierarchy based on properties.**

polygon – a closed plane figure formed from line segments that meet only at their endpoints

quadrilateral – a four-sided polygon

rectangle – a quadrilateral with two pairs of congruent parallel sides and four right angles

rhombus – a parallelogram with all four sides equal in length

square – a parallelogram with four congruent sides and four right angles