MATH

OPERATIONS AND ALGEBRAIC THINKING

Use the four operations with whole numbers to solve problems.

1. **Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.**

$5 \times 8 = 40$

Sally is five years old. Her mom is eight times older. How old is Sally's mom?

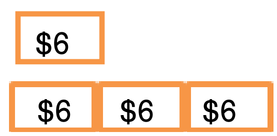
$5 \times 5 = 25$

Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

2. **Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.**

B is the cost of a blue hat in dollars

R is the cost of a red hat in dollars



$3 \times B = R$

$3 \times \$6 = \$18$

3. **Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.**

On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Student: I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pouches would she have left?

$44 \div 6 = p$; $p = 7$ r 2; Mary can fill 7 pouches and have 2 left over.

Gain familiarity with factors and multiples.

Prime vs. Composite:

A prime number is a number greater than 1 that has only 2 factors, 1 and itself. Composite numbers have more than 2 factors.

Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

4. **Find all factor pairs for a whole number in the range 1 – 100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1 – 100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1 – 100 is prime or composite.**

3. Generate and analyze patterns.

Pattern	Rule	Feature(s)
3, 8 13, 18, 23, 28 . . .	Start with 3, add 5	The numbers alternately end with a 3 or 8.
5, 10, 15, 20 . . .	Start with 5, add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that end with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

5. **Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.***

NUMBER AND OPERATIONS IN BASE TEN

Generalize place value understanding for multi-digit whole numbers.

1. **Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.** ***For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.***

In the base ten system, the value of each place is 10 times the value of the place to the immediate right. Because of this, multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left.

How is the 2 in the number 582 similar to and different from the 2 in the number 528?

2. **Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.**

285 = 200 + 80 +5

285 = Two hundred eighty-five

285 = 28 tens plus 5 ones

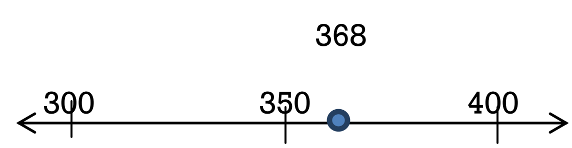
285 = 1 hundred, 18 tens, and 5 ones

3. **Use place value understanding to round multi-digit whole numbers to any place.**

Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student: First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

Round 368 to the nearest hundred.



Use place value understanding and properties of operations to perform multi-digit arithmetic.

Computation algorithm: A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Computation strategy: Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

$$\begin{array}{r} 3892 \\ \underline{+\ 1567} \end{array}$$

Student explanation for this problem:

1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred. (Notates with a 1 above the hundreds column)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (Notates with a 1 above the thousands column)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5. **Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**

$36 \times 94 = (30 + 6) \times (90 + 4)$

$= (30 + 6) \times 90 + (30 + 6) \times 4$

$= 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4$

What would an area array model of 74×38 look like?

	70	4
30	$70 \times 30 = 2{,}100$	$4 \times 30 = 120$
8	$70 \times 8 = 560$	$4 \times 8 = 32$

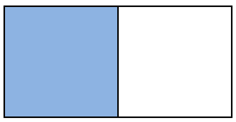
2,100 + 560 + 1,200 + 32 = 2,182

6. **Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.**

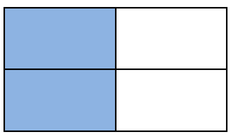
NUMBER AND OPERATION – FRACTIONS

Extend understanding of fraction equivalence and ordering.

1. **Explain why a fraction *a/b* is equivalent to a fraction (*n × a*)/(*n × b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.**



1/2



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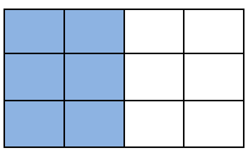
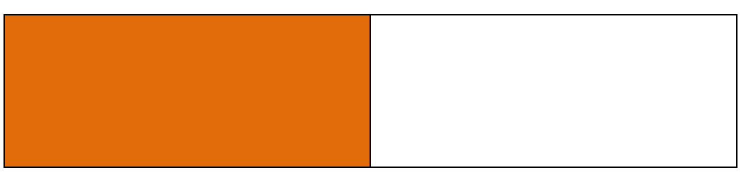


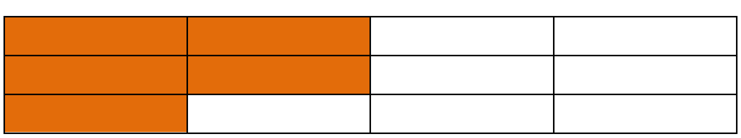
6/12

2. **Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.**

There are two cakes on the counter that are the same size. The first cake has 1/2 of it left. The second cake has 5/12 left. Which cake has more left?

The first cake has more left over. The second cake has 5/12 left which is smaller than 1/2.





Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. **Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*.**

2/3 = 1/3 + 1/3 4/4 + 1/4 = 5/4 5/4 – 3/4 =2/4 or 1/2

a. **Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.**

Mary and Lacey decide to share a pizza. Mary ate 3/6 and Lacey ate 2/6 of the pizza. How much of the pizza did the girls eat together?

Possible solution: The amount of pizza Mary ate can be thought of a 3/6 or 1/6 and 1/6 and 1/6. The amount of pizza Lacey ate can be thought of a 1/6 and 1/6. The total amount of pizza they ate is 1/6 + 1/6 + 1/6 + 1/6 + 1/6 or 5/6 of the whole pizza.

b. **Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.**

3/8 = 1/8 + 1/8 + 1/8 3/8 = 1/8 + 2/8

2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8

5/3 = 3/3 + 2/3 = 1 + 2/3 = 1 2/3

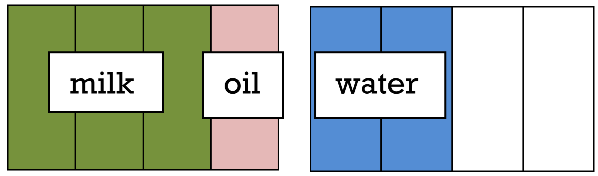
c. **Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.**

Susan and Maria need 8 3/8 of ribbon to package gift baskets. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has 3 1/8 feet of ribbon and Maria has 5 3/8 feet of ribbon. I can write this as 3 1/8 + 5 3/8. I know they have 8 feet of ribbon by adding the 3 and 5. They also have 1/8 and 3/8 which makes a total of 4/8 more. Altogether they have 8 4/8 of ribbon. 8 4/8 is larger than 8 3/8 so they will have enough ribbon to complete the project. They will even have a little extra ribbon left, 1/8 foot.

d. **Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.**

A cake recipe calls for you to use 3/4 cup of milk, 1/4 cup of oil, and 2/4 cup of water. How much liquid was needed to make the cake?

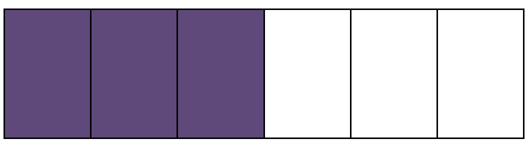


3/4 + 1/4 + 2/4 = 6/4 = 1 2/4

4. **Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.**

   a. **Understand a fraction *a/b* as a multiple of 1/*b*.**
   For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

3/6 = 1/6 + 1/6 + 1/6 = 3 × (1/6)



1/6 2/6 3/6 4/6 5/6 6/6

7/5 = 7 × 1/5 11/3 = 11 × 1/3

   b. **Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.**
   For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)

3 × 2/5 as 2/5 + 2/5+ 2/5 = (3 × 2)/5 = 6/5

In a relay race, each runner runs 1/2 of a lap. If there are 4 team members, how long is the race?

Student draws an area model showing 4 pieces of 1/2 joined together to equal 2.

1/2	1/2

1/2	1/2

If a bucket holds 2 3/4 gallons and 43 buckets of water fill a tank, how much does the tank hold?

The solution 43 × 2 3/4 gallons, one possible way to solve the problem.

$43 \times (2 + 3/4) = 43 \times 11/4 = 473/4 = 118\ 1/4$ gallons

c. **Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.**

For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

5. **Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.**

   For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

	÷ 10	÷ 10		÷ 10
___	___	.	___	___
tens	ones		tenths	hundredths

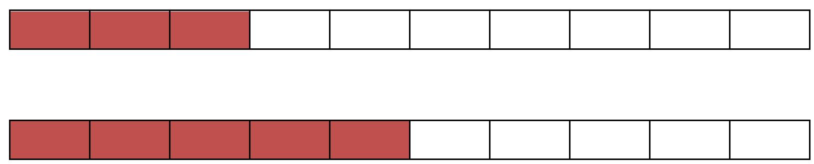
6. **Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.***

$32/100 = 0.32$

Hundreds	Tens	Ones	.	Tenths	Hundredths
			.	3	2

7. **Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.**

$0.3 < 0.5$



MEASUREMENT AND DATA

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

Super- or subordinate unit	Length in terms of basic unit
kilometer	1000 meters
meter	1 meter
centimeter	1/100 meters

Centimeter and meter equivalences

cm	m
100	1
200	2
300	3
500	5
1000	10

Foot and inch equivalences

feet	inches
1	12
2	24
3	36

Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	$n \times 3$

1. **Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb., oz.; l, ml; hr., min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.**
   For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …

Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8 oz., will everyone get at least one glass of milk?

Possible solution:

Charlie plus 10 friends = 11 total people

11 people × 8 ounces (glass of milk) = 88 total ounces

1 quart = 2 pints = 4 cups = 32 ounces

Therefore 1 quart = 2 pints = 4 cups = 32 ounces

2 quarts = 4 pints = 8 cups = 64 ounces

3 quarts = 6 pints = 12 cups = 96 ounces

If Charlie purchased 3 quarts (6 pints) of milk, there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass, then he would have 1 – 8 oz. glass or 1 cup of milk left over.

2. **Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.**

A rectangular garden has an area of 80 square feet. It is 5 feet wide. How long is the garden?

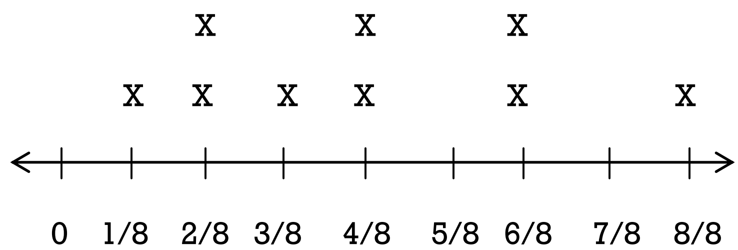
A plan for a house includes a rectangular room with an area of 60 square meters and a perimeter of 32 meters. What are the length and the width of the room?

3. **Apply the area and perimeter formulas for rectangles in real world and mathematical problems.** ***For example, find the width of a rectangular room given the area of the flooring and the length by viewing the area formula as a multiplication equation with an unknown factor.***

Represent and interpret data.

4. **Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.**
For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 inch. They displayed their data collected on a line plot. How many objects measured 1/4 inch? 1/2 inch? If you put all the objects together end to end, what would be the total length of **all** the objects?

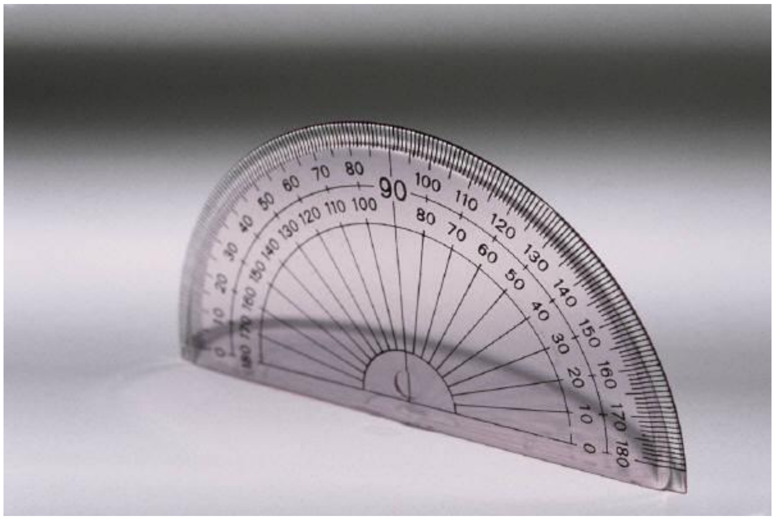


Geometric measurement: understand concepts of angles and measure angles.

An angle

Name	Measurement
Right angle	90˚
Straight angle	180˚
Acute angle	Between 0 and 90˚
Obtuse angle	Between 90˚ and 180˚

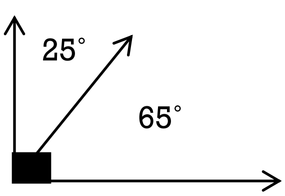
5. **Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:**
   a. **An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle", and can be used to measure angles.**
   b. **An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.**



6. **Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.**

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees, how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses, how many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?



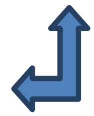
7. **Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.**

GEOMETRY

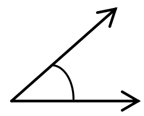
Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. **Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.**

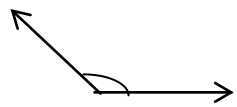
Right angle



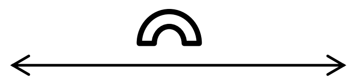
Acute angle



Obtuse angle



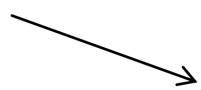
Straight angle



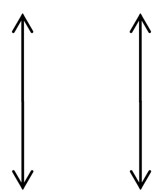
Line segment

Line

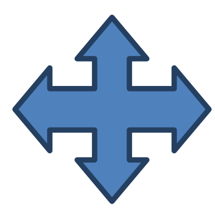
Ray



Parallel lines

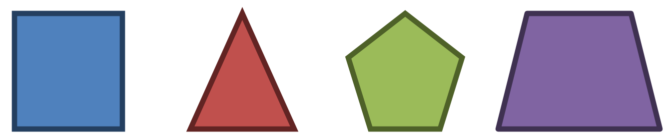


Perpendicular lines



2. **Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.**

Identify which of these shapes have perpendicular or parallel sides and justify your selection.



3. **Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.**

For each figure, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides? Sketch each figure and check your predictions.