MATH

OPERATIONS AND ALGEBRAIC THINKING

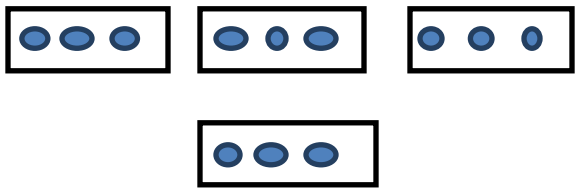
Represent and solve problems using multiplication and division.

1. Interpret products of whole numbers, e.g. interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

For example, describe a context in which a total number of objects can be expressed as 5 × 7.

There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?

There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?



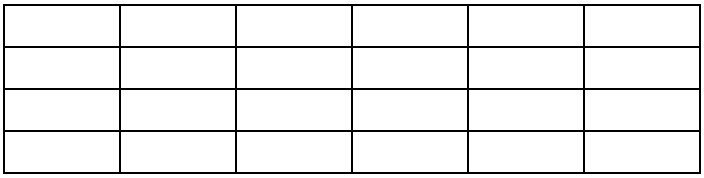
Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 × 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 × 3.

2. Interpret whole-number quotients of whole numbers, e.g. interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3. Use multiplication and division within 100 to solve word problems in situations involveing equal groups, arrays, and measurement quantities, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?



There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class.

$24 = ? \times 6$

$72 \div \Delta = 9$

Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? $3 \times 4 = m$

4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ÷ 3, 6 × 6 = ?

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.

If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative Property of Multiplication)

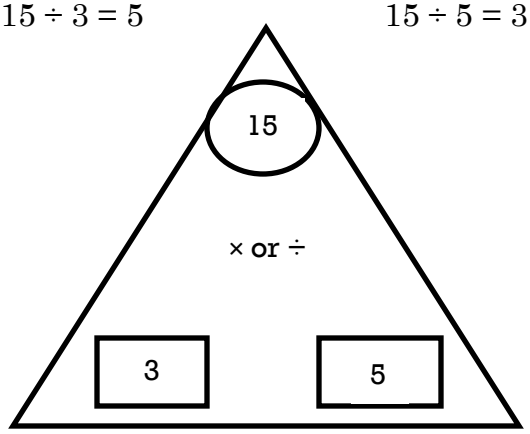
3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative Property of Multiplication)

Knowing that 8 × 5 = 40 and 8 × 2= 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive Property of Multiplication)

0 × 7 = 7 × 0 = 0 (Zero Property of Multiplication)

$3 \times 5 = 15$ $5 \times 3 = 15$

$15 \div 3 = 5$ $15 \div 5 = 3$



6. Understand division as an unknown-factor problem.

For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Strategies students may use to attain fluency include:

Multiplication by zeros and ones

Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)

Tens facts (relating to place value, 5 × 10 is 5 tens or 50)

Five facts (half of tens)

Skip counting (counting groups of __ and knowing how many groups have been counted)

Square numbers (ex. 3 × 3)

Nines (10 groups less one group, e.g. 9 × 3 is 10 groups of 3 minus one group of 3)

Decomposing into known facts (6 × 7 is 6 × 6 plus one more group of 6)

Turn-around facts (Commutative Property)

Fact families (Ex. 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 =

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).

- On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. About how many total miles did they travel?

Student: I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

2, 4, 6, 8, 10, __ , __?

Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).

Multiples of even numbers (2, 4, 6, and 8) are always even numbers.

On a multiplication chart, the products in each row and column increase by the same amount (skip counting).

On an addition chart, the sums in each row and column increase by the same amount.

NUMBER AND OPERATIONS IN BASE 10

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.

23

294

83,342

9,083

There are 178 fourth graders and 225 fifth graders on the playground. What is the total number on the playground?

100 + 200 = 300

70 + 20 = 90

8 + 5 = 13

300 + 90 + 13 = 403 students

2. Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g. 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

50 × 4 = 4 groups of 5 tens

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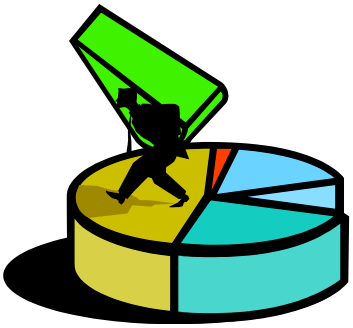
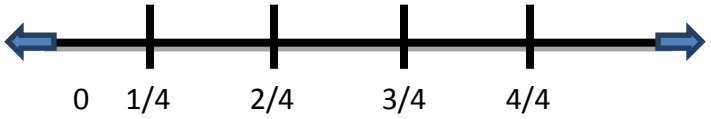
NUMBER AND OPERATION – FRACTIONS

Develop understanding of fractions as numbers.

1. **Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.**





2. **Understand a fraction as a number on the number line; represent fractions on a number line diagram.**

   a. Represent the fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

   b. Represent a fraction *a/b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

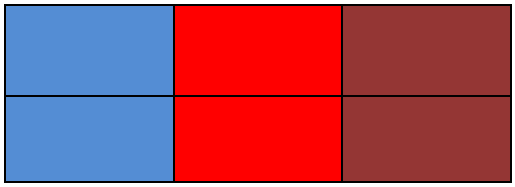
b. Recognize and generate simple equivalent fractions, e.g. 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g. by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g. by using a visual fraction model.

If 6 brownies are shared between 2 people, how many brownies would each person

2/6 = 1/3

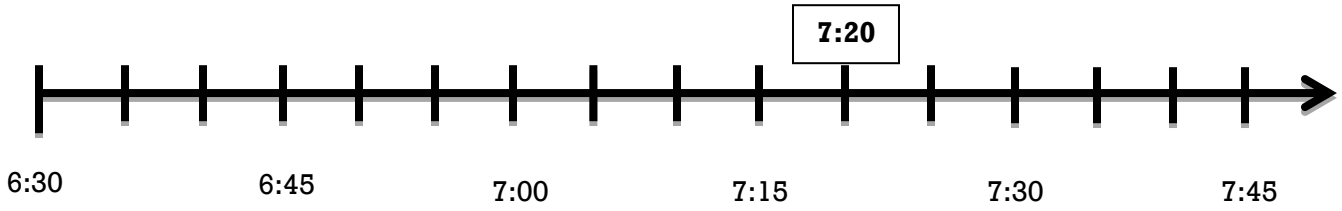


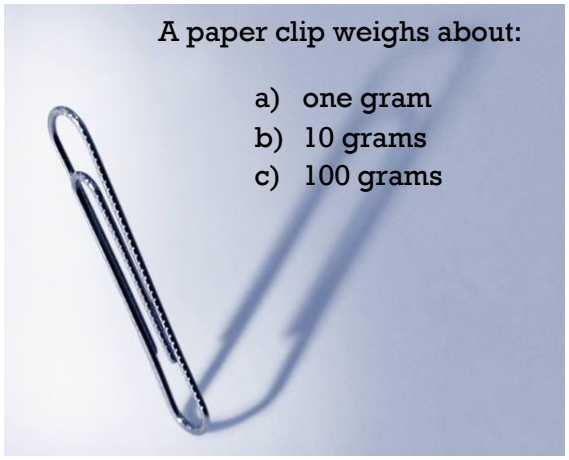
MEASUREMENT AND DATA

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g. by representing the problem on a number line diagram.

Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?





2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g. by using drawings (such as a beaker with a measurement scale) to represent the problem.

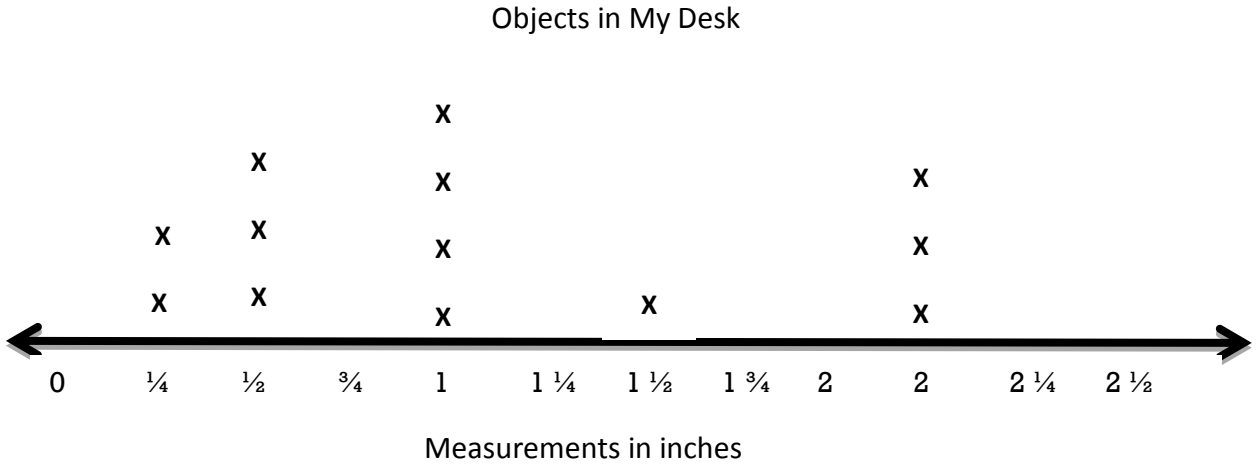
Represent and interpret data.

3. **Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scale bar graphs.**

For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

4. **Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.**

Measure objects in your desk to the nearest ½ or ¼ of an inch. Display data collected on a line plot. How many objects measured ¼? ½? etc . . .

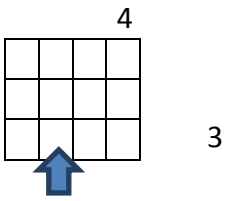


Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square", is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.



One square unit

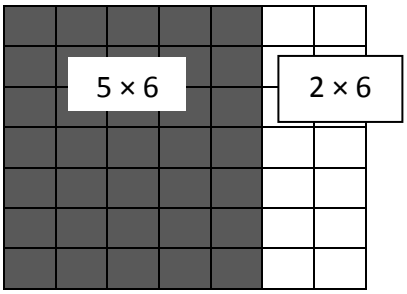
6. Measure areas by counting unit squares (square cm, square m, square in, square ft., and improvised units).

7. Relate area to the operations of multiplication and addition.

Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

8 feet

5 feet



a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

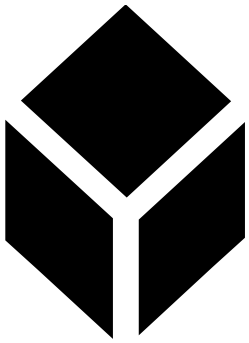
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

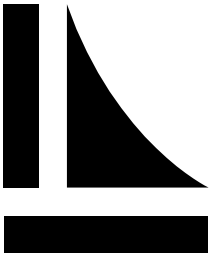
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

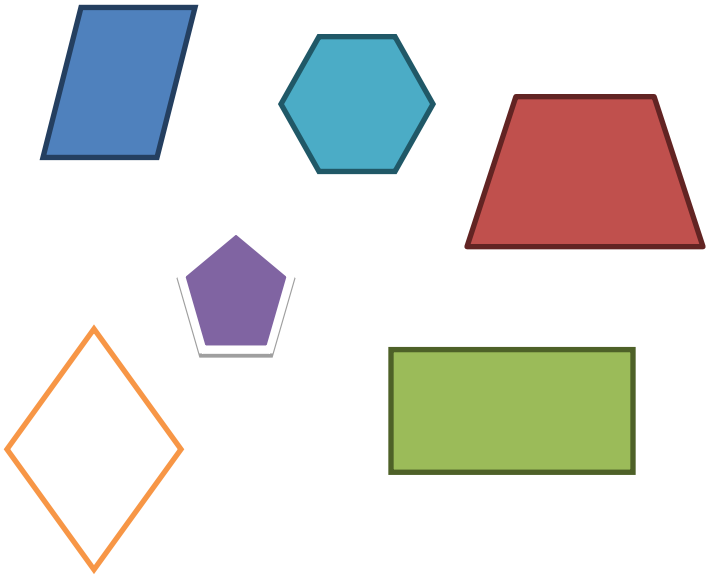
8. **Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.**





GEOMETRY

Reason with shapes and their attributes.



1. **Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides), and that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.**

2. **Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.**

For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.